## 6-6 Rational Exponents

**Write each expression in radical form, or write each radical in exponential form.**

1. $10^{\frac{1}{4}}$

*SOLUTION:*

$10^{\frac{1}{4}} = \sqrt[4]{10}$

2. $x^{\frac{3}{5}}$

*SOLUTION:*

$$\begin{aligned} x^{\frac{3}{5}} &= \left(x^3\right)^{\frac{1}{5}} \\ &= \sqrt[5]{x^3} \end{aligned}$$

3. $\sqrt[3]{15}$

*SOLUTION:*

$\sqrt[3]{15} = 15^{\frac{1}{3}}$

4. $\sqrt[4]{7x^6y^9}$

*SOLUTION:*

$$\begin{aligned} \sqrt[4]{7x^6y^9} &= \left(7x^6y^9\right)^{\frac{1}{4}} \\ &= 7^{\frac{1}{4}}x^{\frac{6}{4}}y^{\frac{9}{4}} \\ &= 7^{\frac{1}{4}}x^{\frac{3}{2}}y^{\frac{9}{4}} \end{aligned}$$

## 6-6 Rational Exponents

**Evaluate each expression.**

5. $343^{\frac{1}{3}}$

*SOLUTION:*

$$\begin{aligned} 343^{\frac{1}{3}} &= \left(7^3\right)^{\frac{1}{3}} \\ &= 7^{\frac{3}{3}} \\ &= 7 \end{aligned}$$

6. $125^{\frac{2}{3}}$

*SOLUTION:*

$$\begin{aligned} 125^{\frac{2}{3}} &= \left(5^3\right)^{\frac{2}{3}} \\ &= 5^2 \\ &= 25 \end{aligned}$$

7. $32^{-\frac{1}{5}}$

*SOLUTION:*

$$\begin{aligned} 32^{-\frac{1}{5}} &= \frac{1}{32^{\frac{1}{5}}} \\ &= \frac{1}{\left(2^5\right)^{\frac{1}{5}}} \\ &= \frac{1}{2} \end{aligned}$$

8. $\dfrac{24}{4^{\frac{3}{2}}}$

*SOLUTION:*

$$\begin{aligned}\frac{24}{4^{\frac{3}{2}}} &= \frac{24}{\left(2^2\right)^{\frac{3}{2}}}\\ &= \frac{24}{2^3}\\ &= \frac{24}{8}\\ &= 3\end{aligned}$$

9. **GARDENING** If the area $A$ of a square is known, then the lengths of its sides $\ell$ can be computed using $\ell = A^{\frac{1}{2}}$. You have purchased a 169 ft$^2$ share in a community garden for the season. What is the length of one side of your square garden?

*SOLUTION:*

Substitute 169 for $A$ in the given equation and simplify.

$$\begin{aligned}l &= 169^{\frac{1}{2}}\\ &= \left(13^2\right)^{\frac{1}{2}}\\ &= 13\end{aligned}$$

The length of one side of the square garden is 13 feet.

**CCSS PRECISION Simplify each expression.**

10. $a^{\frac{3}{4}} \cdot a^{\frac{1}{2}}$

*SOLUTION:*

$$\begin{aligned}a^{\frac{3}{4}} \cdot a^{\frac{1}{2}} &= a^{\frac{3}{4}+\frac{1}{2}}\\ &= a^{\frac{3}{4}+\frac{2}{4}}\\ &= a^{\frac{5}{4}}\end{aligned}$$

11. $\dfrac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}}$

*SOLUTION:*

$$\begin{aligned}\frac{x^{\frac{4}{5}}}{x^{\frac{1}{5}}} &= x^{\frac{4}{5}-\frac{1}{5}} \\ &= x^{\frac{3}{5}}\end{aligned}$$

12. $\dfrac{b^3}{c^{\frac{1}{2}}} \cdot \dfrac{c}{b^{\frac{1}{3}}}$

*SOLUTION:*

$$\begin{aligned}\frac{b^3}{c^{\frac{1}{2}}} \cdot \frac{c}{b^{\frac{1}{3}}} &= b^{3-\frac{1}{3}} c^{1-\frac{1}{2}} \\ &= b^{\frac{8}{3}} c^{\frac{1}{2}}\end{aligned}$$

13. $\sqrt[4]{9g^2}$

*SOLUTION:*

$$\begin{aligned}\sqrt[4]{9g^2} &= \left(9g^2\right)^{\frac{1}{4}} \\ &= \left(3^2 g^2\right)^{\frac{1}{4}} \\ &= 3^{\frac{1}{2}} g^{\frac{1}{2}} \\ &= \sqrt{3g}\end{aligned}$$

14. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{4}}$

*SOLUTION:*

$$\begin{aligned}\frac{\sqrt[5]{64}}{\sqrt[5]{4}} &= \frac{64^{\frac{1}{5}}}{4^{\frac{1}{5}}}\\ &= \frac{\left(2^6\right)^{\frac{1}{5}}}{\left(2^2\right)^{\frac{1}{5}}}\\ &= \frac{2^{\frac{6}{5}}}{2^{\frac{2}{5}}}\\ &= 2^{\frac{6}{5}-\frac{2}{5}}\\ &= 2^{\frac{4}{5}}\\ &= \sqrt[5]{2^4}\\ &= \sqrt[5]{16}\end{aligned}$$

15. $\dfrac{g^{\frac{1}{2}}-1}{g^{\frac{1}{2}}+1}$

*SOLUTION:*

$$\begin{aligned}\frac{g^{\frac{1}{2}}-1}{g^{\frac{1}{2}}+1} &= \frac{\sqrt{g}-1}{\sqrt{g}+1}\\ &= \frac{\sqrt{g}-1}{\sqrt{g}+1}\cdot\frac{\sqrt{g}-1}{\sqrt{g}-1}\\ &= \frac{\left(\sqrt{g}-1\right)^2}{\left(\sqrt{g}\right)^2-1^2}\\ &= \frac{g+1-2\sqrt{g}}{g-1}\\ &= \frac{g+1-2g^{\frac{1}{2}}}{g-1}\end{aligned}$$

**Write each expression in radical form, or write each radical in exponential form.**

16. $8^{\frac{1}{5}}$

*SOLUTION:*

$8^{\frac{1}{5}} = \sqrt[5]{8}$

17. $4^{\frac{2}{7}}$

*SOLUTION:*

$$\begin{aligned} 4^{\frac{2}{7}} &= \left(4^2\right)^{\frac{1}{7}} \\ &= \sqrt[7]{4^2} \\ &= \sqrt[7]{16} \end{aligned}$$

18. $a^{\frac{3}{4}}$

*SOLUTION:*

$$\begin{aligned} a^{\frac{3}{4}} &= \left(a^3\right)^{\frac{1}{4}} \\ &= \sqrt[4]{a^3} \end{aligned}$$

19. $\left(x^3\right)^{\frac{3}{2}}$

*SOLUTION:*

$$\begin{aligned} \left(x^3\right)^{\frac{3}{2}} &= \left(x^9\right)^{\frac{1}{2}} \\ &= \sqrt{x^9} \end{aligned}$$

20. $\sqrt{17}$

*SOLUTION:*

$\sqrt{17} = 17^{\frac{1}{2}}$

21. $\sqrt[4]{63}$

*SOLUTION:*

$\sqrt[4]{63} = 63^{\frac{1}{4}}$

22. $\sqrt[3]{5xy^2}$

*SOLUTION:*

$$\begin{aligned}\sqrt[3]{5xy^2} &= \left(5xy^2\right)^{\frac{1}{3}} \\ &= 5^{\frac{1}{3}} x^{\frac{1}{3}} y^{\frac{2}{3}}\end{aligned}$$

23. $\sqrt[4]{625x^2}$

*SOLUTION:*

$$\begin{aligned}\sqrt[4]{625x^2} &= \left(625x^2\right)^{\frac{1}{4}} \\ &= \left(5^4 x^2\right)^{\frac{1}{4}} \\ &= 5x^{\frac{1}{2}}\end{aligned}$$

**Evaluate each expression.**

24. $27^{\frac{1}{3}}$

*SOLUTION:*

$$\begin{aligned}27^{\frac{1}{3}} &= \left(3^3\right)^{\frac{1}{3}} \\ &= 3\end{aligned}$$

## 6-6 Rational Exponents

25. $256^{\frac{1}{4}}$

*SOLUTION:*

$$\begin{aligned} 256^{\frac{1}{4}} &= \left(4^4\right)^{\frac{1}{4}} \\ &= 4 \end{aligned}$$

26. $16^{-\frac{1}{2}}$

*SOLUTION:*

$$\begin{aligned} 16^{-\frac{1}{2}} &= \frac{1}{16^{\frac{1}{2}}} \\ &= \frac{1}{\left(4^2\right)^{\frac{1}{2}}} \\ &= \frac{1}{4} \end{aligned}$$

27. $81^{-\frac{1}{4}}$

*SOLUTION:*

$$\begin{aligned} 81^{-\frac{1}{4}} &= \frac{1}{81^{\frac{1}{4}}} \\ &= \frac{1}{\left(3^4\right)^{\frac{1}{4}}} \\ &= \frac{1}{3} \end{aligned}$$

28. **CCSS SENSE-MAKING** A women's regulation-sized basketball is slightly smaller than a men's basketball. The radius $r$ of the ball that holds $V$ cubic units of air is $\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.


**a.** Find the radius of a women's basketball.

**b.** Find the radius of a men's basketball.

*SOLUTION:*

**a.** Substitute 413 for $V$ in the expression and simplify.

$$r = \left(\frac{3(413)}{4\pi}\right)^{\frac{1}{3}}$$
$$\approx 4.62$$

The radius of a women's basketball is about 4.62 inches.

**b.** Substitute 455 for $V$ in the expression and simplify.

$$r = \left(\frac{3(455)}{4\pi}\right)^{\frac{1}{3}}$$
$$\approx 4.77$$

The radius of a men's basketball is about 4.77 inches.

29. **GEOMETRY** The radius $r$ of a sphere with volume $V$ is given by $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Find the radius of a ball with a volume of 77 $cm^3$.

*SOLUTION:*
Substitute 77 for $V$ and simplify.

$$\begin{aligned} r &= \left(\frac{3(77)}{4\pi}\right)^{\frac{1}{3}} \\ &\approx 2.64 \end{aligned}$$

The radius of the sphere is about 2.64 cm.

**Simplify each expression.**

30. $x^{\frac{1}{3}} \cdot x^{\frac{2}{5}}$

*SOLUTION:*

$$\begin{aligned} x^{\frac{1}{3}} \cdot x^{\frac{2}{5}} &= x^{\frac{1}{3}+\frac{2}{5}} \\ &= x^{\frac{5+6}{15}} \\ &= x^{\frac{11}{15}} \end{aligned}$$

31. $a^{\frac{4}{9}} \cdot a^{\frac{1}{4}}$

*SOLUTION:*

$$\begin{aligned} a^{\frac{4}{9}} \cdot a^{\frac{1}{4}} &= a^{\frac{4}{9}+\frac{1}{4}} \\ &= a^{\frac{16+9}{36}} \\ &= a^{\frac{25}{36}} \end{aligned}$$

## 6-6 Rational Exponents

32. $b^{-\frac{3}{4}}$

*SOLUTION:*

$$\begin{aligned} b^{-\frac{3}{4}} &= \frac{1}{b^{\frac{3}{4}}} \\ &= \frac{1}{b^{\frac{3}{4}}} \cdot \frac{b^{\frac{1}{4}}}{b^{\frac{1}{4}}} \\ &= \frac{b^{\frac{1}{4}}}{b} \end{aligned}$$

33. $y^{-\frac{4}{5}}$

*SOLUTION:*

$$\begin{aligned} y^{-\frac{4}{5}} &= \frac{1}{y^{\frac{4}{5}}} \cdot \frac{y^{\frac{1}{5}}}{y^{\frac{1}{5}}} \\ &= \frac{y^{\frac{1}{5}}}{y} \end{aligned}$$

34. $\frac{\sqrt[8]{81}}{\sqrt[6]{3}}$

*SOLUTION:*

$$\begin{aligned}\frac{\sqrt[8]{81}}{\sqrt[6]{3}} &= \frac{81^{\frac{1}{8}}}{3^{\frac{1}{6}}}\\ &= \frac{\left(3^4\right)^{\frac{1}{8}}}{3^{\frac{1}{6}}}\\ &= \frac{3^{\frac{1}{2}}}{3^{\frac{1}{6}}}\\ &= 3^{\frac{1}{2}-\frac{1}{6}}\\ &= 3^{\frac{1}{3}}\\ &= \sqrt[3]{3}\end{aligned}$$

35. $\frac{\sqrt[4]{27}}{\sqrt[4]{3}}$

*SOLUTION:*

$$\begin{aligned}\frac{\sqrt[4]{27}}{\sqrt[4]{3}} &= \frac{\left(3^3\right)^{\frac{1}{4}}}{3^{\frac{1}{4}}}\\ &= 3^{\frac{3}{4}-\frac{1}{4}}\\ &= 3^{\frac{1}{2}}\\ &= \sqrt{3}\end{aligned}$$

36. $\sqrt[4]{25x^2}$

*SOLUTION:*

$$\begin{aligned}\sqrt[4]{25x^2} &= \left(5^2 x^2\right)^{\frac{1}{4}}\\ &= 5^{\frac{1}{2}} x^{\frac{1}{2}}\\ &= \sqrt{5x}\end{aligned}$$

37. $\sqrt[6]{81g^3}$

*SOLUTION:*

$$\begin{aligned}\sqrt[6]{81g^3} &= \left(3^4 \cdot g^3\right)^{\frac{1}{6}}\\ &= 3^{\frac{2}{3}} \cdot g^{\frac{1}{2}}\\ &= \sqrt[3]{9} \cdot \sqrt{g}\end{aligned}$$

38. $\dfrac{h^{\frac{1}{2}}+1}{h^{\frac{1}{2}}-1}$

*SOLUTION:*

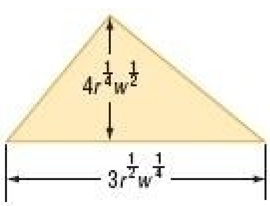
$$\begin{aligned}\frac{h^{\frac{1}{2}}+1}{h^{\frac{1}{2}}-1} &= \frac{\sqrt{h}+1}{\sqrt{h}-1}\\ &= \frac{\sqrt{h}+1}{\sqrt{h}-1} \cdot \frac{\sqrt{h}+1}{\sqrt{h}+1}\\ &= \frac{h+1+2\sqrt{h}}{h-1}\\ &= \frac{h+1+2h^{\frac{1}{2}}}{h-1}\end{aligned}$$

39. $\dfrac{x^{\frac{1}{4}}+2}{x^{\frac{1}{4}}-2}$

*SOLUTION:*

$$\begin{aligned}\frac{x^{\frac{1}{4}}+2}{x^{\frac{1}{4}}-2} &= \frac{\sqrt[4]{x}+2}{\sqrt[4]{x}-2}\\ &= \frac{\sqrt[4]{x}+2}{\sqrt[4]{x}-2}\cdot\frac{\sqrt[4]{x}+2}{\sqrt[4]{x}-2}\\ &= \frac{\sqrt{x}+4+4\sqrt[4]{x}}{\sqrt{x}-4}\\ &= \frac{\sqrt{x}+4+4\sqrt[4]{x}}{\sqrt{x}-4}\cdot\frac{\sqrt{x}+4}{\sqrt{x}+4}\\ &= \frac{x+4\sqrt{x}+4\sqrt{x}+16+4\sqrt[4]{x^3}+16\sqrt[4]{x}}{x-16}\\ &= \frac{x+4x^{\frac{3}{4}}+8x^{\frac{1}{2}}+16x^{\frac{1}{4}}+16}{x-16}\end{aligned}$$

**GEOMETRY** Find the area of each figure.

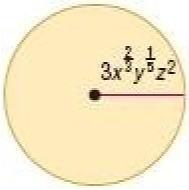
40. 


*SOLUTION:*

The area of a triangle of base *b* and height *h* is $A=\frac{1}{2}bh$.

Substitute $3r^{\frac{1}{2}}w^{\frac{1}{4}}$ and $4r^{\frac{1}{4}}w^{\frac{1}{2}}$ for *b* and *h* and simplify.

The area of the given triangle is $6r^{\frac{3}{4}}w^{\frac{3}{4}}$ unit$^2$.

41. 


*SOLUTION:*

The area of a circle is $A = \pi r^2$.

Substitute $3x^{\frac{2}{3}}y^{\frac{1}{5}}z^2$ for $r$ and simplify.

$$\begin{aligned} A &= \pi\left(3x^{\frac{2}{3}}y^{\frac{1}{5}}z^2\right)^2 \\ &= \pi\left(9x^{\frac{4}{3}}y^{\frac{2}{5}}z^4\right) \\ &\approx 28.27x^{\frac{4}{3}}y^{\frac{2}{5}}z^4 \end{aligned}$$

The area of the given circle is about $28.27x^{\frac{4}{3}}y^{\frac{2}{5}}z^4$ unit$^2$.

42. Find the simplified form of $18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}}$.

*SOLUTION:*

$$\begin{aligned} 18^{\frac{1}{2}} + 2^{\frac{1}{2}} - 32^{\frac{1}{2}} &= \left(3^2 \cdot 2\right)^{\frac{1}{2}} + 2^{\frac{1}{2}} - \left(2^5\right)^{\frac{1}{2}} \\ &= 3\sqrt{2} + \sqrt{2} - 2^2\sqrt{2} \\ &= 3\sqrt{2} + \sqrt{2} - 4\sqrt{2} \\ &= 0 \end{aligned}$$

43. What is the simplified form of $64^{\frac{1}{3}}-32^{\frac{1}{3}}+8^{\frac{1}{3}}$ ?

*SOLUTION:*

$$\begin{aligned}64^{\frac{1}{3}}-32^{\frac{1}{3}}+8^{\frac{1}{3}} &= \left(4^3\right)^{\frac{1}{3}}-\left(2^5\right)^{\frac{1}{3}}+\left(2^3\right)^{\frac{1}{3}}\\ &= 4-2\left(2^2\right)^{\frac{1}{3}}+2\\ &= 6-2(2)^{\frac{2}{3}}\\ &= 6-2\left(2^2\right)^{\frac{1}{3}}\\ &= 6-2\cdot 4^{\frac{1}{3}}\end{aligned}$$

**Simplify each expression.**

44. $a^{\frac{7}{4}}\cdot a^{\frac{5}{4}}$

*SOLUTION:*

$$\begin{aligned}a^{\frac{7}{4}}\cdot a^{\frac{5}{4}} &= a^{\frac{7}{4}+\frac{5}{4}}\\ &= a^{\frac{12}{4}}\\ &= a^3\end{aligned}$$

45. $x^{\frac{2}{3}}\cdot x^{\frac{8}{3}}$

*SOLUTION:*

$$\begin{aligned}x^{\frac{2}{3}}\cdot x^{\frac{8}{3}} &= x^{\frac{2}{3}+\frac{8}{3}}\\ &= x^{\frac{10}{3}}\end{aligned}$$

## 6-6 Rational Exponents

46. $\left(b^{\frac{3}{4}}\right)^{\frac{1}{3}}$

*SOLUTION:*

$$\left(b^{\frac{3}{4}}\right)^{\frac{1}{3}} = b^{\frac{1}{4}}$$

47. $\left(y^{-\frac{3}{5}}\right)^{-\frac{1}{4}}$

*SOLUTION:*

$$\begin{aligned}\left(y^{-\frac{3}{5}}\right)^{-\frac{1}{4}} &= y^{\left(-\frac{3}{5}\right)\times\left(-\frac{1}{4}\right)} \\ &= y^{\frac{3}{20}}\end{aligned}$$

48. $\sqrt[4]{64}$

*SOLUTION:*

$$\begin{aligned}\sqrt[4]{64} &= \sqrt[4]{2^6} \\ &= \left(2^6\right)^{\frac{1}{4}} \\ &= 2^{\frac{3}{2}} \\ &= 2\sqrt{2}\end{aligned}$$

49. $\sqrt[6]{216}$

*SOLUTION:*

$$\begin{aligned}\sqrt[6]{216} &= \sqrt[6]{6^3} \\ &= \left(6^3\right)^{\frac{1}{6}} \\ &= 6^{\frac{1}{2}} \\ &= \sqrt{6}\end{aligned}$$

50. $d^{-\frac{5}{6}}$

*SOLUTION:*

$$\begin{aligned} d^{-\frac{5}{6}} &= \frac{1}{d^{\frac{5}{6}}} \cdot \frac{d^{\frac{1}{6}}}{d^{\frac{1}{6}}} \\ &= \frac{d^{\frac{1}{6}}}{d} \end{aligned}$$

51. $w^{-\frac{7}{8}}$

*SOLUTION:*

$$\begin{aligned} w^{-\frac{7}{8}} &= \frac{1}{w^{\frac{7}{8}}} \cdot \frac{w^{\frac{1}{8}}}{w^{\frac{1}{8}}} \\ &= \frac{w^{\frac{1}{8}}}{w} \end{aligned}$$

52. **WILDLIFE** A population of 100 deer is reintroduced to a wildlife preserve. Suppose the population does extremely well and the deer population doubles in two years. Then the number *D* of deer after *t* years is given by $D = 100 \cdot 2^{\frac{t}{2}}$.

**a.** How many deer will there be after $4\frac{1}{2}$ years?

**b.** Make a table that charts the population of deer every year for the next five years.

**c.** Make a graph using your table.

**d.** Using your table and graph, decide whether this is a reasonable trend over the long term. Explain.

*SOLUTION:*

**a.** Substitute 4.5 for *t* and simplify.

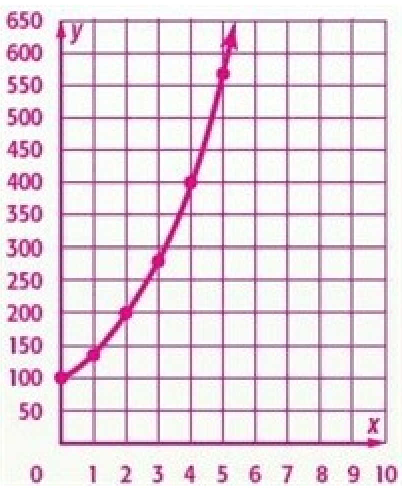
$$\begin{aligned} D &= 100 \cdot 2^{\frac{4.5}{2}} \\ &\approx 100 \cdot 4.75 \\ &= 475 \end{aligned}$$

There will be about 475 deer after $4\frac{1}{2}$ years.

**b.** Substitute 0, 1, 2, 3, 4 and 5 for *t* and make the table.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $D = 100 \cdot 2^{\frac{t}{2}}$ | $D = 100 \cdot 2^{\frac{0}{2}}$ $= 100$ | $D = 100 \cdot 2^{\frac{1}{2}}$ $\approx 141$ | $D = 100 \cdot 2^{\frac{2}{2}}$ $= 200$ | $D = 100 \cdot 2^{\frac{3}{2}}$ $\approx 282$ | $D = 100 \cdot 2^{\frac{4}{2}}$ $= 400$ | $D = 100 \cdot 2^{\frac{5}{2}}$ $\approx 565$ |

**c.** Plot the points on a coordinate plane and connect them as shown.




**d.** Sample answer: No; it is not reasonable to think that the population will continue to grow without bounds. This does not take into account the death rate of the deer.

## 6-6 Rational Exponents

**Simplify each expression.**

53. $\dfrac{f^{-\frac{1}{4}}}{4f^{\frac{1}{2}} \cdot f^{-\frac{1}{3}}}$

*SOLUTION:*

$$\begin{aligned}\frac{f^{-\frac{1}{4}}}{4f^{\frac{1}{2}} \cdot f^{-\frac{1}{3}}} &= \frac{f^{\frac{1}{3}}}{4f^{\frac{1}{2}}.f^{\frac{1}{4}}}\\ &= \frac{f^{\frac{1}{3}}}{4f^{\frac{3}{4}}}\\ &= \frac{f^{\frac{1}{3}}}{4f^{\frac{3}{4}}} \cdot \frac{f^{\frac{1}{4}}}{f^{\frac{1}{4}}}\\ &= \frac{f^{\frac{7}{12}}}{4f}\end{aligned}$$

54. $\dfrac{g^{\frac{5}{2}}}{g^{\frac{1}{2}} + 2}$

*SOLUTION:*

$$\begin{aligned}\frac{g^{\frac{5}{2}}}{g^{\frac{1}{2}} + 2} &= \frac{g^{\frac{5}{2}}}{g^{\frac{1}{2}} + 2} \cdot \frac{g^{\frac{1}{2}} - 2}{g^{\frac{1}{2}} - 2}\\ &= \frac{g^{\frac{6}{2}} - 2g^{\frac{5}{2}}}{\left(g^{\frac{1}{2}}\right)^2 - 2^2}\\ &= \frac{g^3 - 2g^{\frac{5}{2}}}{g - 4}\end{aligned}$$

55. $\dfrac{c^{\frac{2}{3}}}{c^{\frac{1}{6}}}$

*SOLUTION:*

$$\begin{aligned}\frac{c^{\frac{2}{3}}}{c^{\frac{1}{6}}} &= \frac{c^{\frac{2}{3}}}{c^{\frac{1}{6}}} \cdot \frac{c^{\frac{5}{6}}}{c^{\frac{5}{6}}}\\ &= \frac{c^{\frac{9}{6}}}{c}\\ &= \frac{c \cdot c^{\frac{3}{6}}}{c}\\ &= c^{\frac{1}{2}}\end{aligned}$$

56. $\dfrac{z^{\frac{4}{5}}}{z^{\frac{1}{2}}}$

*SOLUTION:*

$$\begin{aligned}\frac{z^{\frac{4}{5}}}{z^{\frac{1}{2}}} &= \frac{z^{\frac{4}{5}}}{z^{\frac{1}{2}}} \cdot \frac{z^{\frac{1}{2}}}{z^{\frac{1}{2}}}\\ &= \frac{z^{\frac{13}{10}}}{z}\\ &= \frac{z \cdot z^{\frac{3}{10}}}{z}\\ &= z^{\frac{3}{10}}\end{aligned}$$

57. $\sqrt{23} \cdot \sqrt[3]{23^2}$

*SOLUTION:*

$$\begin{aligned}\sqrt{23} \cdot \sqrt[3]{23^2} &= (23)^{\frac{1}{2}}(23)^{\frac{2}{3}} \\ &= 23^{\frac{7}{6}} \\ &= 23 \cdot 23^{\frac{1}{6}} \\ &= 23\sqrt[6]{23}\end{aligned}$$

58. $\sqrt[8]{36h^4 j^4}$

*SOLUTION:*

$$\begin{aligned}\sqrt[8]{36h^4 j^4} &= \left(6^2 h^4 j^4\right)^{\frac{1}{8}} \\ &= 6^{\frac{1}{4}} h^{\frac{1}{2}} j^{\frac{1}{2}}\end{aligned}$$

59. $\sqrt{\sqrt{81}}$

*SOLUTION:*

$$\begin{aligned}\sqrt{\sqrt{81}} &= \left(\sqrt{81}\right)^{\frac{1}{2}} \\ &= \left(81^{\frac{1}{2}}\right)^{\frac{1}{2}} \\ &= 81^{\frac{1}{4}} \\ &= \left(3^4\right)^{\frac{1}{4}} \\ &= 3\end{aligned}$$

## 6-6 Rational Exponents

60. $\sqrt[4]{\sqrt{256}}$

*SOLUTION:*

$$\begin{aligned}\sqrt[4]{\sqrt{256}} &= \left(\sqrt{256}\right)^{\frac{1}{4}} \\ &= \left(256^{\frac{1}{2}}\right)^{\frac{1}{4}} \\ &= 256^{\frac{1}{8}} \\ &= \left(2^8\right)^{\frac{1}{8}} \\ &= 2\end{aligned}$$

61. $\frac{ab}{\sqrt{c}}$

*SOLUTION:*

$$\begin{aligned}\frac{ab}{\sqrt{c}} &= \frac{ab}{\sqrt{c}} \cdot \frac{\sqrt{c}}{\sqrt{c}} \\ &= \frac{ab\sqrt{c}}{c}\end{aligned}$$

62. $\frac{xy}{\sqrt[3]{z}}$

*SOLUTION:*

$$\begin{aligned}\frac{xy}{\sqrt[3]{z}} &= \frac{xy}{\sqrt[3]{z}} \cdot \frac{\sqrt[3]{z^2}}{\sqrt[3]{z^2}} \\ &= \frac{xy\sqrt[3]{z^2}}{z}\end{aligned}$$

63. $\dfrac{8^{\frac{1}{6}}-9^{\frac{1}{4}}}{\sqrt{3}+\sqrt{2}}$

*SOLUTION:*

$$\begin{aligned}\frac{8^{\frac{1}{6}}-9^{\frac{1}{4}}}{\sqrt{3}+\sqrt{2}} &= \frac{\left(2^3\right)^{\frac{1}{6}}-\left(3^2\right)^{\frac{1}{4}}}{\sqrt{3}+\sqrt{2}}\cdot\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\\ &= \frac{2^{\frac{1}{2}}-3^{\frac{1}{2}}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\\ &= \frac{\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{3}-\sqrt{2}\right)}{3-2}\\ &= -\left(3+2-2\sqrt{6}\right)\\ &= 2\sqrt{6}-5\end{aligned}$$

64. $\dfrac{x^{\frac{5}{3}}-x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}}+z^{\frac{2}{3}}}$

*SOLUTION:*

$$\begin{aligned}\frac{x^{\frac{5}{3}}-x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}}+z^{\frac{2}{3}}} &= \frac{x^{\frac{5}{3}}-x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}}+z^{\frac{2}{3}}}\cdot\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\\ &= \frac{x^2-x^{\frac{2}{3}}z^{\frac{4}{3}}}{x+x^{\frac{1}{3}}z^{\frac{2}{3}}}\\ &= \frac{\left(x+x^{\frac{1}{3}}z^{\frac{2}{3}}\right)\left(x-x^{\frac{1}{3}}z^{\frac{2}{3}}\right)}{\left(x+x^{\frac{1}{3}}z^{\frac{2}{3}}\right)}\\ &= x-x^{\frac{1}{3}}z^{\frac{2}{3}}\end{aligned}$$

## 6-6 Rational Exponents

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the functions $f(x) = x^3$ and $g(x) = x^{\frac{1}{3}}$.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| −2 | | |
| −1 | | |
| 0 | | |
| 1 | | |
| 2 | | |

**a. TABULAR** Copy and complete the table to the right.

**b. GRAPHICAL** Graph $f(x)$ and $g(x)$.

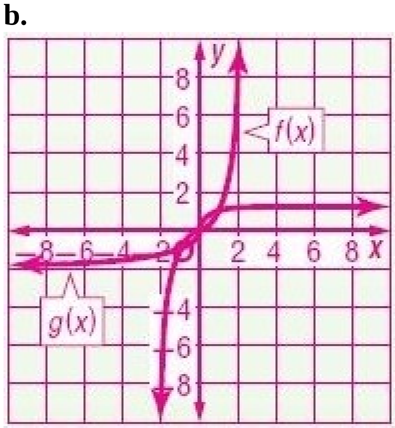
**c. VERBAL** Explain the transformation between $f(x)$ and $g(x)$.

*SOLUTION:*

**a.** Substitute –2, –1, 0, 1 and 2 for $x$ in the function $f(x)$ and $g(x)$ respectively and complete the table.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| −2 | −8 | −1.26 |
| −1 | −1 | −1 |
| 0 | 0 | 0 |
| 1 | 1 | 1 |
| 2 | 8 | 1.26 |

**b.**




**c.** It is a reflection of the line $y = x$.

66. **REASONING** Determine whether $-x^{-2} = (-x)^{-2}$ is *always, sometimes,* or *never* true. Explain your reasoning.

*SOLUTION:*
The statement is never true.
The quantities are not the same. When the negative is enclosed inside of the parentheses and the base is raised to an even power, the answer is positive. When the negative is not enclosed inside of the parentheses and the base is raised to an even power, the answer is negative.

67. **CHALLENGE** Consider $\sqrt[4]{(-16)^3}$.

**a.** Explain why the expression is not a real number.

**b.** Find $n$ such that $n\sqrt[4]{(-16)^3}$ is a real number.

*SOLUTION:*
a. Sample answer: $\sqrt[4]{(-16)^3} = \sqrt[4]{-4096}$

There is no real number that when raised to the forth power results in a negative number.

b. Sample answer: $\sqrt[4]{-1}$
This is much like multiplying $\sqrt{-2} \cdot \sqrt{-18}$. The product rule of radicals is only defined for real numbers, so you must first rewrite each square root before multiplying as follows: $i\sqrt{2} \cdot i\sqrt{18}$. Then you can multiply the square roots and complex numbers separately. This will result in the product $i^2 \cdot \sqrt{36}$ or –6. From two complex numbers we obtained a product that is a real number. In a similar manner multiplying $\sqrt[4]{(-16)^3}$ by $\sqrt[4]{-1}$ actually produces multiple products of which two are real number products 8 and –8 and the other two are complex products 8***i*** and –8***i***. This multiplication will be explained in a future course.

68. **OPEN ENDED** Find two different expressions that equal 2 in the form $x^{\frac{1}{a}}$.

*SOLUTION:*
Sample answer: Since $x^{\frac{1}{a}} = \sqrt[a]{x}$ and the square root of 4 is 2 and the 4th root of 16 is 2: $4^{\frac{1}{2}}$ and $16^{\frac{1}{4}}$ each equal 2.

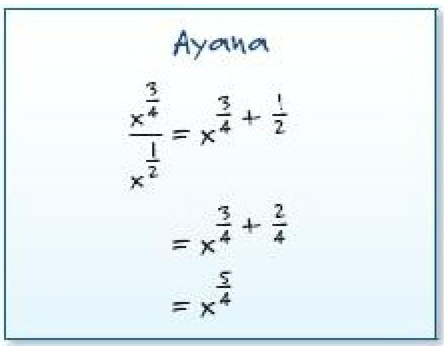
## 6-6 Rational Exponents

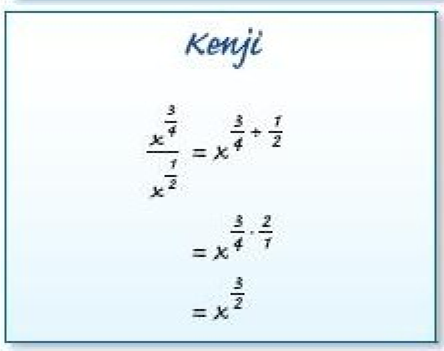
69. **WRITING IN MATH** Explain how it might be easier to simplify an expression using rational exponents rather than using radicals.

*SOLUTION:*

Sample answer: It may be easier to simplify an expression when it has rational exponents because all the properties of exponents apply. We do not have as many properties dealing directly with radicals. However, we can convert all radicals to rational exponents, and then use the properties of exponents to simplify.

70. **CCSS CRITIQUE** Ayana and Kenji are simplifying $\frac{x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$. Is either of them correct? Explain your reasoning.




*SOLUTION:*

No.

Ayana added the exponents and Kenji divided the exponents. The exponents should have been subtracted.

71. The expression $\sqrt{56-c}$ is equivalent to a positive integer when $c$ is equal to

**A** 8

**B** –8

**C** 56

**D** 36

*SOLUTION:*
When $c$ = –8, the radicand $56-c$ becomes a perfect square.

Therefore, option B is the correct answer.

72. **SAT/ACT** Which of the following sentences is true about the graphs of $y=2(x-3)^2+1$ and $y=2(x+3)^2+1$?
**F** Their vertices are maximums.

**G** The graphs have the same shape with different vertices.

**H** The graphs have different shapes with the same vertices.

**J** The graphs have different shapes with different vertices.

**K** One graph has a vertex that is a maximum while the other graph has a vertex that is a minimum.

*SOLUTION:*
Compare with the general equation for a parabola in vertex form.

$$y=a(x-h)+k$$

Since $a$ = 2 in both equations, which is positive, the graph both open upwards, and vertices are both minimums.

The only number which is changed is $h$. So, the graphs have the same shape, but the $x$-coordinate of the vertex is different.
Option G is the correct answer.

73. **GEOMETRY** What is the converse of the statement? *If it is summer, then it is hot outside*.

**A** If it is not hot outside, then it is not summer.

**B** If it is not summer, then it is not hot outside.

**C** If it is hot outside, then it is summer.

**D** If it is hot outside, it is not summer.

*SOLUTION:*
The converse is produced by interchanging the hypothesis and the conclusion.

Option C is the correct answer.

74. **SHORT RESPONSE** If $3^5 \cdot p = 3^3$, then find $p$.

*SOLUTION:*

$$\begin{aligned} 3^5 \cdot p &= 3^3 \\ 3^5 \cdot 3^{-5} \cdot p &= 3^3 \cdot 3^{-5} \\ p &= 3^{-2} \end{aligned}$$

**Simplify.**

75. $\sqrt{243}$

*SOLUTION:*

$$\begin{aligned} \sqrt{243} &= \sqrt{81 \cdot 3} \\ &= \sqrt{9^2 \cdot 3} \\ &= 9\sqrt{3} \end{aligned}$$

76. $\sqrt[3]{16y^3}$

*SOLUTION:*

$$\begin{aligned} \sqrt[3]{16y^3} &= \sqrt[3]{2^4 y^3} \\ &= 2y\sqrt[3]{2} \end{aligned}$$

77. $3\sqrt[3]{56y^6z^3}$

*SOLUTION:*

$$\begin{aligned}3\sqrt[3]{56y^6z^3} &= 3\sqrt[3]{8\cdot 7\left(y^2\right)^3 z^3}\\ &= 3\sqrt[3]{2^3\cdot 7\left(y^2\right)^3 z^3}\\ &= 3\cdot 2\cdot y^2 z\sqrt[3]{7}\\ &= 6y^2 z\sqrt[3]{7}\end{aligned}$$

78. **PHYSICS** The speed of sound in a liquid is $s=\sqrt{\frac{B}{d}}$, where *B* is the bulk modulus of the liquid and d is its density. For water, $B = 2.1\times 10^9$ N/m$^2$ and $d = 10^3$ kg/m$^3$. Find the speed of sound in water to the nearest meter per second.

*SOLUTION:*

Substitute $2.1\times 10^9$ and $10^3$ for *B* and *d* and simplify.

$$\begin{aligned}s &= \sqrt{\frac{2.1\times 10^9}{10^3}}\\ &= \sqrt{2.1\times 10^6}\\ &= \sqrt{2100000}\\ &\approx 1449\end{aligned}$$

The speed of the sound in water is 1449 m/s.

**Find *p* (–4) and *p* (*x* + *h*) for each function.**

79. $p(x) = x - 2$

*SOLUTION:*

Substitute –4 for *x* and simplify.

$$\begin{aligned}p(-4) &= -4-2\\ &= -6\end{aligned}$$

Substitute *x* + *h* for *x* and simplify.

$$p(x+h) = x+h-2$$

80. $p(x) = -x + 4$

*SOLUTION:*
Substitute –4 for *x* and simplify.

$$\begin{aligned} p(-4) &= -(-4) + 4 \\ &= 4 + 4 \\ &= 8 \end{aligned}$$

Substitute $x + h$ for *x* and simplify.

$$\begin{aligned} p(x+h) &= -(x+h) + 4 \\ &= -x - h + 4 \end{aligned}$$

81. $p(x) = 6x + 3$

*SOLUTION:*
Substitute –4 for *x* and simplify.

$$\begin{aligned} p(-4) &= 6(-4) + 3 \\ &= -24 + 3 \\ &= -21 \end{aligned}$$

Substitute $x + h$ for *x* and simplify.

$$\begin{aligned} p(x+h) &= 6(x+h) + 3 \\ &= 6x + 6h + 3 \end{aligned}$$

82. $p(x) = x^2 + 5$

*SOLUTION:*
Substitute –4 for *x* and simplify.

$$\begin{aligned} p(-4) &= (-4)^2 + 5 \\ &= 16 + 5 \\ &= 21 \end{aligned}$$

Substitute $x + h$ for *x* and simplify.

$$\begin{aligned} p(x+h) &= (x+h)^2 + 5 \\ &= x^2 + 2xh + h^2 + 5 \end{aligned}$$

83. $p(x) = x^2 - x$

*SOLUTION:*
Substitute –4 for *x* and simplify.

$$\begin{aligned} p(-4) &= (-4)^2 - (-4) \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

Substitute $x + h$ for *x* and simplify.

$$\begin{aligned} p(x+h) &= (x+h)^2 - (x+h) \\ &= x^2 + 2xh + h^2 - x - h \end{aligned}$$

84. $p(x)=2x^3-1$

*SOLUTION:*
Substitute –4 for *x* and simplify.

$$\begin{aligned}p(-4)&=2(-4)^3-1\\&=2(-64)-1\\&=-128-1\\&=-129\end{aligned}$$

Substitute $x + h$ for *x* and simplify.

$$\begin{aligned}p(x+h)&=2(x+h)^3-1\\&=2\left(x^3+3x^2h+3xh^2+h^3\right)-1\\&=2x^3+6x^2h+6xh^2+2h^3-1\end{aligned}$$

**Solve each equation by factoring.**

85. $x^2-11x=0$

*SOLUTION:*

$$\begin{aligned}x^2-11x&=0\\x(x-11)&=0\end{aligned}$$

$$\begin{array}{ccc}x-11=0 & \text{or} & x=0\\ x=11 & \text{or} & x=0\end{array}$$

The solutions are 0 and 11.

86. $x^2 + 6x - 16 = 0$

*SOLUTION:*

$$x^2 + 6x - 16 = 0$$
$$(x+8)(x-2) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x + 8 = 0$$
$$x = 2 \quad \text{or} \quad x = -8$$

The solutions are –8 and 2.

87. $4x^2 - 13x = 12$

*SOLUTION:*

$$4x^2 - 13x = 12$$
$$4x^2 - 13x - 12 = 0$$
$$4x^2 - 16x + 3x - 12 = 0$$
$$4x(x-4) + 3(x-4) = 0$$
$$(4x+3)(x-4) = 0$$

$$x - 4 = 0 \quad \text{or} \quad 4x + 3 = 0$$
$$x = 4 \quad \text{or} \quad x = -\frac{3}{4}$$

The solutions are $-\frac{3}{4}$ and 4. .

88. $x^2 - 14x = -49$

*SOLUTION:*

$$x^2 - 14x = -49$$
$$x^2 - 14x + 49 = 0$$
$$(x-7)(x-7) = 0$$
$$x - 7 = 0$$
$$x = 7$$

The solution is 7.

89. $x^2 + 9 = 6x$

*SOLUTION:*

$$\begin{aligned} x^2 + 9 &= 6x \\ x^2 + 9 - 6x &= 0 \\ (x-3)^2 &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

The solution is 3.

90. $x^2 - 3x = -\frac{9}{4}$

*SOLUTION:*

$$x^2 - 3x = -\frac{9}{4}$$

$$x^2 - 3x + \frac{9}{4} = 0$$

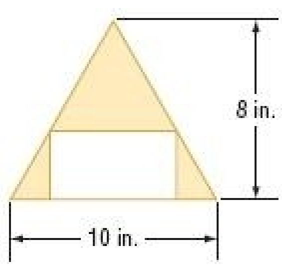
$$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right) = 0$$

$$x - \frac{3}{2} = 0$$

$$x = \frac{3}{2}$$

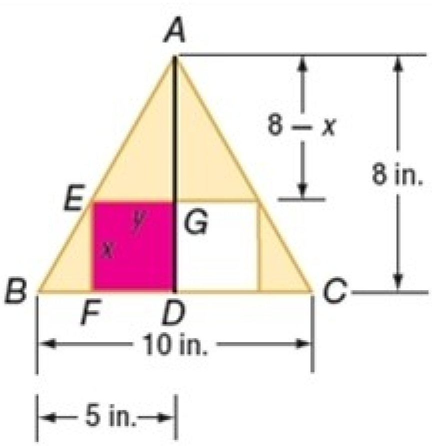
The solution is $x = \frac{3}{2}$.

91. **GEOMETRY** A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (Hint: Use similar triangles.)



*SOLUTION:*

Draw an altitude and name the vertices. Let $x$ and $y$ be the width and the length of the shaded region.




The triangle $ABD$ and the triangle $AEG$ are similar triangle. So, $\frac{AG}{EG} = \frac{AD}{BD}$.

$$\frac{8-x}{y} = \frac{8}{5}$$
$$y = \frac{40-5x}{8}$$

The area of the shaded region is $A = x\left(\frac{40-5x}{8}\right) = \frac{40x-5x^2}{8}$.

The function gets maximum at $x = 4$.

Substitute the 4 for $x$ and find the value of $y$.

$$y = \frac{40-5(4)}{8} = 2.5$$

Therefore, the length of the rectangle is $2y = 5$ in.

The dimensions of the rectangle with maximum area are 5 inches by 4 inches.

**Find each power.**

92. $\left(\sqrt{x-3}\right)^2$

*SOLUTION:*

$$\begin{aligned}\left(\sqrt{x-3}\right)^2 &= \left((x-3)^{\frac{1}{2}}\right)^2 \\ &= x-3\end{aligned}$$

93. $\left(\sqrt[3]{3x-4}\right)^3$

*SOLUTION:*

$$\begin{aligned}\left(\sqrt[3]{3x-4}\right)^3 &= \left((3x-4)^{\frac{1}{3}}\right)^3 \\ &= 3x-4\end{aligned}$$

94. $\left(\sqrt[4]{7x-1}\right)^4$

*SOLUTION:*

$$\begin{aligned}\left(\sqrt[4]{7x-1}\right)^4 &= \left((7x-1)^{\frac{1}{4}}\right)^4 \\ &= 7x-1\end{aligned}$$

95. $\left(\sqrt{x}-4\right)^2$

*SOLUTION:*

$$\begin{aligned}\left(\sqrt{x}-4\right)^2 &= \left(\sqrt{x}\right)^2 - 8\sqrt{x} + (4)^2 \\ &= x - 8\sqrt{x} + 16\end{aligned}$$

## 6-6 Rational Exponents

96. $\left(2\sqrt{x}-5\right)^2$

*SOLUTION:*

$$\begin{aligned}\left(2\sqrt{x}-5\right)^2 &= \left(2\sqrt{x}\right)^2 - 20\sqrt{x} + (5)^2 \\ &= 4x - 20\sqrt{x} + 25\end{aligned}$$

97. $\left(3\sqrt{x}+1\right)^2$

*SOLUTION:*

$$\begin{aligned}\left(3\sqrt{x}+1\right)^2 &= \left(3\sqrt{x}\right)^2 + 6\sqrt{x} + 1^2 \\ &= 9x + 6\sqrt{x} + 1\end{aligned}$$